EXTENSIONS ON ADAPTIVE OUTPUT FEEDBACK CONTROL OF NONLINEAR DYNAMICAL SYSTEMS USING NEURAL NETWORKS

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Abstract— This paper reviews a recently developed adaptive output feedback control methodology using neural networks. It is shown how to remove the contraction mapping assumption used in the original paper and it is clarified the impact of this removal in the approximate model selection. Moreover, it is illustrated how to explore the previous information, available in the form of an approximate model, to improve the controlled system performance.

Keywords— Output feedback, neural networks, adaptive control, input-output linearization.

Resumo— Neste trabalho faz-se uma revisão de uma técnica recentemente desenvolvida de controle adaptativo utilizando realimentação de saída e redes neurais. É mostrado como se pode remover a suposição de mapeamento tipo contração utilizada no trabalho original, além de se esclarecer o impacto deste resultado na seleção de um modelo aproximado da planta. Ilustra-se também como se pode explorar a informação prévia disponível, sob a forma de um modelo aproximado, na melhoria de desempenho do sistema controlado.

Palavras-chave— Realimentação de saída, redes neurais, controle adaptativo, linearização entrada-saída.

1 Introduction

Research on adaptive control using neural networks (NN) is motivated by novel actuator devices and the existence of a large class of nonlinear systems for which a systematic constructive control procedure has not been developed yet.

The first works in adaptive control using neural networks date back 1980s and early 1990s. A pioneer work in this field that used NN in identification and control of nonlinear dynamical systems is that reported by Narendra and Parthasarathy (1990). Following Narendra and Parthasarathy's work, Sanner and Slotine (1992) used radial basis function (RBF) NN to control affine nonlinear systems by state feedback. Furthermore, Sanner and Slotine combined NN with proportional-derivative (PD) and nonlinear sliding mode controllers and proved system stability through a Lyapunov based analysis. A similar approach was used by Lewis et al. (1995) to control a planar robotic manipulator. As in the work of Sanner and Slotine, and in contrast to Narendra and Parthasarathy's work, this approach does not need an off-line training phase and the control architecture was composed of linear in the parameters neural networks.

All the works that were mentioned so far are characterized by NN that operates in conjunction with other linear or nonlinear controllers and are thus a kind of assisted control techniques. A different approach that uses NN as the main centralized controller can be found in the works of Rovithakis (1999), Ge et al. (1999) and Ge and Zhang (2003). The main drawback shared by these approaches lies on the centralized control architecture that can lead to undesirable or completely intolerable transient behavior, specially in critical applications like aircraft control (Stevens and Lewis, 2003).

An approach that can use prior knowledge about the plant and combines input-output linearization (Isidori, 1995), NN and linear control is reported in (Calise et al., 2001). In this paper NN compensates the inversion error that arises when an approximate model is used in dynamic inversion. A contraction mapping is assumed between the adaptive control and the inversion error, which poses some restrictions in the model selection.

Based in the work of Hovakimyan et al. (2004), which is similar but uses observers to calculate the error vector, we remove the contraction mapping assumption and show the impact of this result in model selection. It is shown that the same update law, proposed by Calise et al. (2001), can be used without the contraction mapping assumption provided that some additional knowledge about plant dynamics is available. This article also address the problem regarding the model choice in a way to satisfy the conditions used in the stability proof.

2 Problem Statement

Consider a SISO, continuous-time and observable nonlinear dynamical system given by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u), \tag{1}$$

$$y = h(\boldsymbol{x}), \tag{2}$$

where $\boldsymbol{x} \in \Omega \subset \mathcal{R}^n$ is the state, $u \in \mathcal{R}$ is the input and $y \in \mathcal{R}$ is the output. In (1)-(2) $\boldsymbol{f}(.,.) : (\mathcal{R}^n \times \mathcal{R}) \mapsto \mathcal{R}$ and $h(.) : \mathcal{R}^n \mapsto \mathcal{R}$ are both continuous and differentiables and n need not be known.

Assumption 1. The dynamical system given by Eqs. (1)-(2) is feedback linearizable with relative degree r. That is

$$y = h(\boldsymbol{x}) = L_f^0 h(\boldsymbol{x}) = h_0(\boldsymbol{x})$$

$$\dot{y} = \left(\frac{\partial h}{\partial \boldsymbol{x}}\right)^T \boldsymbol{f}(\boldsymbol{x}, u) = L_f h(\boldsymbol{x}) = h_1(\boldsymbol{x})$$

$$\ddot{y} = \left(\frac{\partial L_f h(x)}{\partial \boldsymbol{x}}\right)^T \boldsymbol{f}(\boldsymbol{x}, u) = L_f^2 h(\boldsymbol{x}) = h_2(\boldsymbol{x})$$

$$\vdots$$

$$y^{(r)} = L_f^r h(\boldsymbol{x}, u) = h_r(\boldsymbol{x}, u), \tag{3}$$

with $\frac{dh_k}{du} = 0$ for $0 \leq k < r$.

As in (Calise et al., 2001), it is desired that the output y follows y_d generated by a stable reference model and is also assumed that only the output measurements are available. Moreover, any internal dynamics is assumed stable.

The control strategy is based in input-output linearization and is shown in section 3.

3 Controller Architecture

Input-output linearization is done using a *pseudo control* variable defined as follows

$$y^{(r)} = v^*, \tag{4}$$

$$v^* = h_r(\boldsymbol{x}, u) \tag{5}$$

and

$$u^* = h_r^{-1}(\boldsymbol{x}, v^*). \tag{6}$$

An approximate inversion can be made with

$$v = \hat{h}_r(y, u), \tag{7}$$

$$u = \hat{h}_r^{-1}(y, v). \tag{8}$$

In (Calise et al., 2001), the output of the NN is designed to cancel Δ defined by

$$\Delta = \Delta(\boldsymbol{x}, u) = h_r(\boldsymbol{x}, u) - \hat{h}_r(y, u) \tag{9}$$

and v is given by

$$v = y_d^{(r)} + v_{dc} - v_{ad}, \tag{10}$$

where v_{dc} is the output of a single-input multi-output (SIMO) linear compensator, and v_{ad} is the output of a NN. In this way, v_{ad} is designed to cancel Δ, which depends of v_{ad} through v. To guarantee existence and uniqueness of a solution for v_{ad} a contraction mapping between it and Δ is assumed and the following two conditions restrict the choice of an approximate model:

$$\text{sgn}(\partial h_r/\partial u) = \text{sgn}(\partial \hat{h}_r/\partial u), \tag{11}$$

$$||\partial \hat{h}_r/\partial u|| > \frac{||\partial h_r/\partial u||}{2} > 0. \tag{12}$$

To eliminate the need of condition (12), based in the work (Hovakimyan et al., 2004), we define

$$v_a = v_{dc} + y_d^{(r)} \tag{13}$$

and

$$v_b = \hat{h}_r(y, h_r^{-1}(\boldsymbol{x}, v_a)). \tag{14}$$

Because of the assumed invertibility of $\hat{h}_r(.,.)$ and $h_r(.,.)$ with respect to the second argument

$$h_r^{-1}(\boldsymbol{x}, v_a) = \hat{h}_r^{-1}(y, v_b)$$

$$v_a = h_r(\boldsymbol{x}, \hat{h}_r^{-1}(y, v_b)). \tag{15}$$

Now, based on Eqs. (9),(10), (14) and (15) we can write

$$\begin{aligned} v_{ad} - \Delta &= v_{ad} - h_r(\boldsymbol{x}, u) + \hat{h}_r(y, u) \\ &= v_{ad} - h_r(\boldsymbol{x}, u) + v_a - v_{ad} \\ &= -h_r(\boldsymbol{x}, u) + v_a \\ &= -h_r(\boldsymbol{x}, \hat{h}_r^{-1}(y, v)) + v_a \\ &= h_r(\boldsymbol{x}, \hat{h}_r^{-1}(y, v_b)) - h_r(\boldsymbol{x}, \hat{h}_r^{-1}(y, v)) \end{aligned} \tag{16}$$

If h_r is continuous and differentiable over the interval $[v, v_b]$, then, by applying the mean value theorem, there exists a $\bar{v}$ such that

$$\frac{dh_r}{dv}|_{v=\bar{v}} = \frac{h_r(\boldsymbol{x}, \hat{h}_r^{-1}(y, v_b)) - h_r(\boldsymbol{x}, \hat{h}_r^{-1}(y, v))}{v_b - v} \tag{17}$$

where

$$\bar{v} = v + \theta(v_b - v), \qquad 0 \leq \theta \leq 1. \tag{18}$$

Equation (16) can now be rewritten as

$$\begin{aligned} v_{ad} - \Delta &= \tfrac{dh_r}{dv}|_{v=\bar{v}}(v_b - v) \\ &= h_{\bar{v}}(v_b - v) \\ &= h_{\bar{v}}(\hat{h}_r(y, h_r(\boldsymbol{x}, v_a)) - v_a + v_{ad}) \\ &= h_{\bar{v}}(v_{ad} - \bar{\Delta}(\boldsymbol{x}, y, v_a)), \end{aligned} \tag{19}$$

where

$$\bar{\Delta}(\boldsymbol{x}, y, v_a) = \hat{h}_r(y, h_r(\boldsymbol{x}, v_a)) - v_a, \tag{20}$$

$$h_{\bar{v}} = \frac{dh_r}{dv}|_{v=\bar{v}} \tag{21}$$

According to **Assumption 1**, $\frac{\partial h_r}{\partial u} \neq 0$ and $\frac{\partial u}{\partial v} = \left(\frac{\partial v}{\partial u}\right)^{-1}$, therefore, in the light of (7) we have

$$\tfrac{dh_r}{dv} = \tfrac{\partial h_r}{\partial u}\left(\tfrac{\partial \hat{h}_r}{\partial u}\right)^{-1}. \tag{22}$$

If condition (11) is satisfied, then

$$\frac{dh_r}{dv} = \frac{\partial h_r}{\partial u}\left(\frac{\partial \hat{h}_r}{\partial u}\right)^{-1} > 0 \tag{23}$$

and the error dynamics can be expressed as

$$\tilde{y}^{(r)} = -v_{dc} + h_{\bar{v}}(v_{ad} - \bar{\Delta}). \tag{24}$$

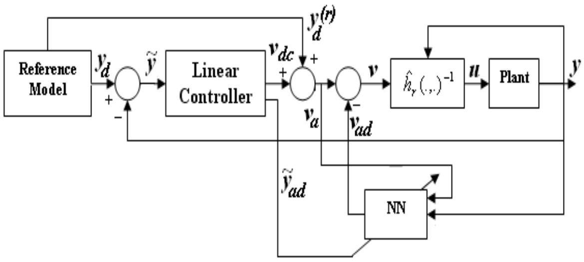

Figure 1: Control System block diagram.

Now, the adaptive signal v_{ad} is designed to cancel $\bar{\Delta}$, which does not depend explicitly on u.

Figure 1 shows a schematic diagram of the controller architecture. It should be noted that Fig.1 is similar to the controller structure in (Calise et al., 2001), but now the input to the neural network is not v but v_a. For systems with stable zero dynamics an additional delayed input $v(t-d)$ may be required to account for the unobservable states. The delayed v signal avoids the fixed point solution problem at the expense of increased NN approximate error bound (Kim, 2003).

The SIMO linear controller is given by

$$\begin{bmatrix} v_{dc}(s) \\ \tilde{y}_{ad}(s) \end{bmatrix} = \frac{1}{D_{dc}(s)} \begin{bmatrix} N_{dc}(s) \\ N_{ad}(s) \end{bmatrix} \tilde{y}(s). \quad (25)$$

where

$$\tilde{y} = y_d - y. \quad (26)$$

Assumption 2. The roots of $D_{dc}(s)$ are located at the open left half complex plane.

The transfer function between $\tilde{y}$ and $h_{\bar{v}}(v_{ad} - \bar{\Delta})$ is given by

$$\tilde{y}(s) = \frac{D_{dc}(s)[h_{\bar{v}}(v_{ad} - \bar{\Delta})](s)}{s^{(r)}D_{dc}(s) + N_{dc}(s)} \quad (27)$$

Therefore, from Eqs. (25) and (27) the transfer function between $\tilde{y}_{ad}$ and $h_{\bar{v}}(v_{ad} - \bar{\Delta})$ is given by

$$\begin{aligned} \tilde{y}_{ad}(s) &= \tfrac{N_{ad}(s)[h_{\bar{v}}(v_{ad}-\bar{\Delta})](s)}{s^{(r)}D_{dc}(s)+N_{dc}(s)} \\ &= G(s)[h_{\bar{v}}(v_{ad} - \bar{\Delta})](s). \end{aligned} \quad (28)$$

A stable low pass filter $T^{-1}(s)$ is introduced to make the transfer function between $\tilde{y}_{ad}$ and $[h_{\bar{v}}(v_{ad} - \bar{\Delta})]$ strictly positive real (SPR):

$$\tilde{y}_{ad}(s) = \bar{G}(s)T^{-1}(s)[h_{\bar{v}}(v_{ad} - \bar{\Delta})](s), \quad (29)$$

where $\bar{G}(s) = G(s)T(s)$.

A linear in the parameters NN can be used to approximate the inversion error $\bar{\Delta}$. The output of such a network is given by

$$y_{nn} = W^T\phi(\boldsymbol{x}), \quad (30)$$

where W are the NN weights and $\phi(.)$ is a vector basis function over the domain of approximation.

For a general continuous and k times differentiable function $g(\boldsymbol{x})$ with $\boldsymbol{x} \in \mathcal{D} \subset \mathcal{R}^n$

$$g(\boldsymbol{x}) = W^T\phi(\boldsymbol{x}) + \epsilon(\boldsymbol{x}). \quad (31)$$

In Eq. (31), $\epsilon(\boldsymbol{x})$ is the reconstruction error.

Definition: The functional range of a NN that have its output given by (30) is dense over a compact domain $x \in \mathcal{D}$ if for any continuous and k times differentiable $g(.)$ and ϵ^* there exists a finite set of bounded weights W such that Eq. (31) holds with $||\epsilon(\boldsymbol{x})|| < \epsilon^*$.

Theorem 1 *(Calise et al., 2001) Given $\epsilon^* > 0$, there exists a set of bounded weights W such that $\Delta(\boldsymbol{x}, y, v_a)$ can be approximated over a compact domain $\mathcal{D} \subset \Omega \times R$ by a linearly parameterized neural network*

$$\bar{\Delta} = W^T\phi(\eta) + \epsilon(\eta), ||\epsilon|| < \epsilon^* \quad (32)$$

using the input vector

$$\eta(t) = [1 \quad \bar{v}_d^T \quad \bar{y}_d^T]^T, \quad (33)$$

$$\begin{aligned} \bar{v}_d^T &= [v_a(t) \;\; v_a(t-d)... \quad v_a(t-(n_1-r-1)d)]^T, \\ \bar{y}_d^T &= [y(t) \;\; y(t-d)... \quad y(t-(n_1-1)d)]^T \end{aligned} \quad (34)$$

with $n_1 \geq n$ and $d > 0$, provided there exists a suitable basis of activation functions $\phi(.)$ on the compact domain $\mathcal{D}$.

Proof: See (Calise et al., 2001). □

Theorem 1 allow Eq. (29) to be rewritten as

$$\begin{aligned} \tilde{y}_{ad}(s) &= \bar{G}(s)T^{-1}(s)[h_{\bar{v}}(\hat{W}^T\phi - W^T\phi - \epsilon)](s) \\ &= \bar{G}(s)T^{-1}(s)[h_{\bar{v}}(\tilde{W}^T\phi - \epsilon)](s) \\ &= \bar{G}(s)\{h_{\bar{v}}\tilde{W}^T\phi_f(s) + \delta(s) - \epsilon_f(s)\} \end{aligned} \quad (35)$$

where

$$\begin{aligned} &T^{-1}(s)(h_{\bar{v}}\tilde{W}^T\phi)(s) = T^{-1}(s)(h_{\bar{v}}\tilde{W}^T\phi)(s) + \\ &h_{\bar{v}}\tilde{W}^T\phi_f(s) - h_{\bar{v}}\tilde{W}^T\phi_f(s) \\ &T^{-1}(s)(h_{\bar{v}}\tilde{W}^T\phi)(s) = h_{\bar{v}}\tilde{W}^T\phi_f(s) + \delta(s), \end{aligned} \quad (36)$$

$$\delta(s) = T^{-1}(s)(h_{\bar{v}}\tilde{W}^T\phi)(s) - h_{\bar{v}}\tilde{W}^T\phi_f(s), \quad (37)$$

$$\phi_f(s) = T^{-1}(s)\phi(s), \quad (38)$$

and

$$\epsilon_f(s) = T^{-1}(s)[h_{\bar{v}}\epsilon](s). \quad (39)$$

Provided that $h_{\bar{v}}$ is continuous over the interval $[v_a, v]$, it reaches its maximum value h_{vMAX} on this interval and if ϕ is a squashing function, an upper bound for δ can be written as

$$||\delta|| \leq c||\tilde{W}||_F h_{vMAX}. \quad (40)$$

The following update law (Calise et al., 2001) can be used to adjust the NN free parameters

$$\dot{\hat{W}} = -F[\tilde{y}_{ad}\phi_f + \lambda\hat{W}], \quad (41)$$

where F is a positive definite matrix and λ is the adaptation gain.

Next section shows how the relaxation of the contraction mapping assumption alters the stability proof.

4 Stability Proof

Let $\{A_c, B_c, C_c\}$, $\{A_f, B_f, C_f\}$ be the controller canonical state space realization of $\bar{G}(s)$ and all the cast filters $T^{-1}(s)$ used to filter ϕ, respectively and z and z_f be the corresponding state variables associated with these realizations. Since the filter $T^{-1}(s)$ is stable and $\bar{G}(s)$ is SPR, it follows that

$$A_f^T P_f + P_f A_f = -Q_f, \quad (42)$$

$$A_c^T P + P A_c = -Q \quad (43)$$

and

$$P B_c = C_c^T, \quad (44)$$

for positive definite Q_f, P_f, Q and P. Equation (44) follows from the Kalman-Yakuvovich lemma (Astrom and Wittenmark, 1995).

The following theorem is the main result of this paper and assures ultimate boundedness of the closed loop signals.

Theorem 2 *Subject to Assumptions (1-2) and if* $\mathrm{sgn}(\frac{\partial h_r}{\partial u}) = \mathrm{sgn}(\frac{\partial \hat{h}_r}{\partial u})$, *the error signals of the system comprised of the dynamics in Eq. (1-2), together with the dynamics associated with the realization of the controller and the NN adaptation rule, are uniformly ultimately bounded, provided that the following conditions hold:*

$$\bar{Q}_m > 2||C_c||^2 \quad (45)$$

and

$$\lambda > \bar{c}^2/4, \quad (46)$$

where

$$\bar{Q}_m = \frac{Q_m}{h_{vMAX}} - H_v P_{MAX} \quad (47)$$

and

$$\bar{c} = c\frac{h_{vMAX}}{h_{vMIN}}. \quad (48)$$

In Eqs. (47-48) Q_m *is the minimum eigenvalue of* Q, P_{MAX} *is the maximum eigenvalue of* P, h_{vMIN} *is the minimum value of* h_v *and* H_v *is the maximum value of* $\frac{d}{dt}(\frac{1}{h_{\bar{v}}})$.

Proof: Consider the following positive definite decrescent function

$$V = \frac{1}{2h_{\bar{v}}} z^T P z + \frac{1}{2} z_f^T P z_f + \frac{1}{2}\tilde{W}^T F^{-1}\tilde{W}. \quad (49)$$

Differentiating (49) with respect to time gives

$$\begin{aligned}
\dot{V} &= \tfrac{1}{2}\left[\tfrac{d}{dt}\left(\tfrac{1}{h_{\bar{v}}}\right) z^T P z + \tfrac{1}{h_{\bar{v}}}(Pz + P^T z)^T \dot{z}\right] \\
&+ \tfrac{1}{2}\left[P_f z_f + P_f^T z_f\right]^T \dot{z}_f \\
&+ \tfrac{1}{2}\left[(F^{-1})^T \tilde{W} + F^{-1}\tilde{W}\right]^T \dot{\tilde{W}} \\
\dot{V} &= \tfrac{1}{2}\tfrac{d}{dt}\left(\tfrac{1}{h_{\bar{v}}}\right) z^T P z + \tfrac{1}{2h_{\bar{v}}}(Pz + P^T z)^T [A_c z + \\
&+ B_c(h_{\bar{v}}\tilde{W}^T \phi_f + \delta - \epsilon_f)] \\
&+ \tfrac{1}{2}\left(P_f z_f + P_f^T z_f\right)^T (A_f z_f + B_f \phi) \\
&+ \tfrac{1}{2}\left[(F^{-1})^T \tilde{W} + F^{-1}\tilde{W}\right]^T \dot{\tilde{W}}.
\end{aligned} \quad (50)$$

Because of the symmetry of the matrices F^{-1}, P and P_f, using the fact that the transposition of a scalar is equal to the same scalar, and substituting the update law (41) and (42-44) in (50) we get

$$\begin{aligned}
\dot{V} &= \left[\tfrac{d}{dt}\left(\tfrac{1}{h_{\bar{v}}}\right)\tfrac{z^T P z}{2}\right] - \tfrac{z^T Q z}{2h_{\bar{v}}} \\
&+ \tfrac{z^T C_c^T}{h_{\bar{v}}}\left(h_{\bar{v}}\tilde{W}^T\phi_f + \delta - \epsilon_f\right) - \tfrac{z_f^T Q_f z_f}{2} \\
&+ z^T P_f B_f \phi + \tilde{W}^T F^{-1}\dot{\hat{W}}
\end{aligned}$$

$$\begin{aligned}
\dot{V} &= \left[\tfrac{d}{dt}\left(\tfrac{1}{h_{\bar{v}}}\right)\tfrac{z^T P z}{2}\right] - \tfrac{z^T Q z}{2h_{\bar{v}}} + \tilde{y}_{ad}\tilde{W}^T\phi_f \\
&+ \tfrac{\tilde{y}_{ad}}{h_{\bar{v}}}(\delta - \epsilon_f) - \tfrac{z_f^T Q_f z_f}{2} + z^T P_f B_f \phi \\
&- \tilde{y}_{ad}\tilde{W}^T\phi_f - \tilde{W}^T\lambda\hat{W}
\end{aligned}$$

$$\begin{aligned}
\dot{V} &= -\tfrac{z^T Q z}{2h_{\bar{v}}} + \tfrac{\tilde{y}_{ad}}{h_{\bar{v}}}(\delta - \epsilon_f) - \tfrac{z_f^T Q_f z_f}{2} \\
&+ z_f^T P_f B_f \phi - \lambda\tilde{W}^T(W + \tilde{W}) + \left[\tfrac{d}{dt}\left(\tfrac{1}{h_{\bar{v}}}\right)\tfrac{z^T P z}{2}\right].
\end{aligned} \quad (51)$$

Assuming that the filter $T^{-1}(s)$ is scaled so that its maximum gain is unity, the right side of Eq. (51) can be upper bounded as

$$\begin{aligned}
\dot{V} &\le -\tfrac{Q_m||z||^2}{2h_{vMAX}} - \tfrac{Q_{fm}||z_f||^2}{2} + \tfrac{H_v P_{MAX}||z||^2}{2} \\
&+ ||z_f||||P_f B_f||||\phi|| + \tfrac{c||\tilde{y}_{ad}||||\tilde{W}||_F h_{vMAX}}{h_{vMIN}} \\
&+ \tfrac{||\tilde{y}_{ad}||\epsilon^* h_{vMAX}}{h_{vMIN}} - \lambda||\tilde{W}||(||\tilde{W}||_F - W^*)
\end{aligned}$$

$$\begin{aligned}
\dot{V} &\le -\tfrac{Q_m||\tilde{y}_{ad}||^2}{2||Cc||^2 h_{vMAX}} + \tfrac{H_v P_{MAX}||\tilde{y}_{ad}||^2}{2||Cc||^2} \\
&+ \bar{c}||\tilde{y}_{ad}||||\tilde{W}||_F + ||\tilde{y}_{ad}||\bar{\epsilon}^* \\
&- ||z_f||\left(\tfrac{Q_{fm}||z_f||}{2} - ||P_f B_f||||\phi||\right) \\
&- \lambda||\tilde{W}||(||\tilde{W}||_F - W^*)
\end{aligned}$$

$$\begin{aligned}
\dot{V} &\le -\tfrac{\bar{Q}_m||\tilde{y}_{ad}||^2}{2||Cc||^2} + \bar{c}||\tilde{y}_{ad}||||\tilde{W}||_F + ||\tilde{y}_{ad}||\bar{\epsilon}^* \\
&- ||z_f||\left(\tfrac{Q_{fm}||z_f||}{2} - ||P_f B_f||||\phi||\right) \\
&- \lambda||\tilde{W}||(||\tilde{W}||_F - W^*),
\end{aligned} \quad (52)$$

where

$$\bar{\epsilon}^* = \frac{\epsilon^* h_{vMAX}}{h_{vMIN}} \ge 0. \quad (53)$$

Inequality (52) is the same as inequality (40) of (Calise et al., 2001) with Q_m, ϵ^* and c replaced by $\bar{Q}_m$, $\bar{\epsilon}^*$ and $\bar{c}$ respectively. Therefore, the remaining of the proof is straightforward. □

Remark 1. As can be seen in (52), in (Calise et al., 2001) there is an error which was carried out throughout the stability proof. Therefore, the corrected condition that restricts the choice of **Q** is $Q_m > 2||C_{c1}||^2$.

Remark 2. For systems with unknown but constant control effectiveness, $H_v = 0$ and condition (45) becomes

$$Q_m > 2h_{vMAX}||C_c||^2. \quad (54)$$

The same condition can be used as an approximation to systems with unknown and slowing varying $h_{\bar{v}}$ ($\frac{d}{dt}\left\{\frac{1}{h_{\bar{v}}}\right\} \simeq 0$).

Section 5 shows a design example to clarify how this result affects the inversion model choice and the adaptive control design.

5 Design Example

Most of papers that employ techniques similar to that presented in works like (Calise et al., 2001; Hovakimyan et al., 2002; Hovakimyan et al., 2004), assume that contraction mapping holds and use an approximate inverting model given by

$$v = u. \quad (55)$$

Sometimes, an approximate linear model around an operating point is used. This occurs specially in the state feedback case (Rysdyk and Calise, 2005). In both cases, the question that arises about the existence of a contraction mapping between v_{ad} and Δ is not touched upon. The following example shows that the contraction mapping assumption can be easily violated in both cases.

Example: Consider the nonlinear system given by

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= -x_1^3 + cos(x_2) + (K_1 + K_2 sin(\mu x_1))u, \end{aligned} \quad (56)$$

with $\mu = 0.01$, $K_1 = 1.2$ and $K_2 = 1$. Linearization around the operating point $\boldsymbol{x}_{op} = [-15 \quad 0]^T$ furnishes an approximate control effectiveness $\frac{\partial \hat{h}_r}{\partial u} \simeq 1.0506$, which clearly violates the contraction mapping assumption if $x_1 = \pi/2\mu$. The same thing occurs if Eq. (55) is used in the inversion.

Suppose that an inverting control law of the form $v = \bar{K}_1 u$ is used, then $\frac{\partial \hat{h}_r}{\partial u} = \bar{K}_1$ and

$$h_{\bar{v}} = \frac{K_1 + K_2 sin(\mu x_1)}{\bar{K}_1}|_{v=\bar{v}}, \quad (57)$$

$$h_{vMAX} = \frac{K_1 + K_2}{\bar{K}_1}. \quad (58)$$

On the other hand

$$H_v = max\left\{\frac{d}{dt}\left[\frac{\bar{K}_1}{K_1 + K_2 sin(\mu x_1)}\right]\right\}, \quad (59)$$

$$H_v = \mu \bar{K}_1 K_2 max\left\{\frac{-cos(\mu x_1)x_2}{[K_1 + K_2 sin(\mu x_1)]^2}\right\}. \quad (60)$$

This suggests that an increasing in $\bar{K}_1$ decreases $h_{\bar{v}}$ and increases H_v, therefore this might not be the best way to assure a solution for (45) and (47). Now suppose that $v = [\bar{K}_1 + \bar{K}_2 sin(\mu x_1)]u$. Carrying out the same calculations for h_{vMAX} and H_v we get

$$h_{vMAX} = max\left\{\frac{K_1 + K_2 sin(\mu x_1)}{\bar{K}_1 + \bar{K}_2 sin(\mu x_1)}\right\}, \quad (61)$$

$$H_v = max\left\{\frac{\mu(K_1\bar{K}_2 - K_2\bar{K}_1)cos(\mu x_1)x_2}{[K_1 + K_2 sin(\mu x_1)]^2}\right\}. \quad (62)$$

If we have, $K_1 = 1.2$, $\bar{K}_1 = 1.2$, $K_2 = 1$ and $max(x_2) = 1$, for the case of linear inversion $v = \bar{K}_1 u$, $h_{vMAX} \simeq 1.83$ and $H_{vMAX} \simeq 0.06$. On the other hand, for the nonlinear inverting control law with $\bar{K}_1 = 0.8$ and $\bar{K}_2 = 0.6$, $h_{vMAX} \simeq 1.57$ and $H_v \simeq 0.004$. Therefore, an approximate nonlinear inverting control law, that uses only output measurements, can reduce simutaneously h_{vMAX} and H_v more than a linear inverting control law that has a perfect estimate of K_1. Another possibility to decrease the eigenvalues of P and to reduce $||C_c||$ is to change the magnitude of the eigenvalues of A_c but in this case the system performance may experience some degradation.

Now suppose that $v = \bar{K}_1 u$, with $\bar{K}_1 = 0.5$, and that the system has to follow a second order reference model given by

$$\ddot{y}_d = -y_d - \sqrt{1.8}\dot{y}_d + u, \quad (63)$$

commanded by a square reference signal of amplitude equal to 2 and period $40s$.

To place the closed loop of the input-output linearized error dynamics at $p_0 = p_1 = -4$, $p_2 = -1 + j$ and $p_3 = -1 - j$, the following linear lead-lag compensator with an approximate integral control action was used

$$\begin{bmatrix} v_{dc}(s) \\ \tilde{y}_{ad}(s) \end{bmatrix} = \begin{bmatrix} \frac{33.9s^2+48s+32.48}{s^2+10s+0.0999} \\ \frac{0.17s^2+0.34s+0.25}{s^2+10s+0.0999} \end{bmatrix} \tilde{y}(s) \quad (64)$$

and the stable low pass filter was set to

$$T^{-1}(s) = \frac{1}{5.86s + 1}. \quad (65)$$

Remark 3. A lead-lag compensator without integral control action leads to a system response with steady state error. On the other hand, a pure proportional-integral compensator can not be used because it violates **Assumption 2**, therefore, a compensator of the form $G_{PI}(s) = K_p + \frac{K_I}{s+\theta}$, with $\theta = 0.01$, was added to the lead-lag controller.

Remark 4. The reader can check that this design easily satisfies (45) and (47), when $H_v = -0.003$, $h_{vMAX} = 2.44$ and $Q = I_6$. These bounds were calculated supposing that y and $\dot{y}$ follow the reference model. After the design was carried out, the estimated signal ranges were verified through simulations to assure that the design conditions were fully satisfied. It is important to point out that in a practical situation the system may be unknown or partially known. Therefore H_v and h_{vMAX} should be estimated during some system identification procedure.

The Gaussian NN comprises six units, with centers randomly distributed over the interval $[-0.5\ 2.5]$, $\sigma^2 = 6$ and the network input is $\eta(t) = [v_l\ y(t)\ y(t-0.02)\ y(t-0.04)\ y(t-0.06)\ 1]^T$. The adaptation gains are $F = 500I_6$ and $\lambda = 0.0015$, where I_6 is the identity matrix of order 6.

Simulation results are shown in Fig. 2, where it can be seen that the neural network can compensate the inversion error and the system has a significant performance improvement without loss of stability, even when the contraction mapping assumption is violated over the signal range.

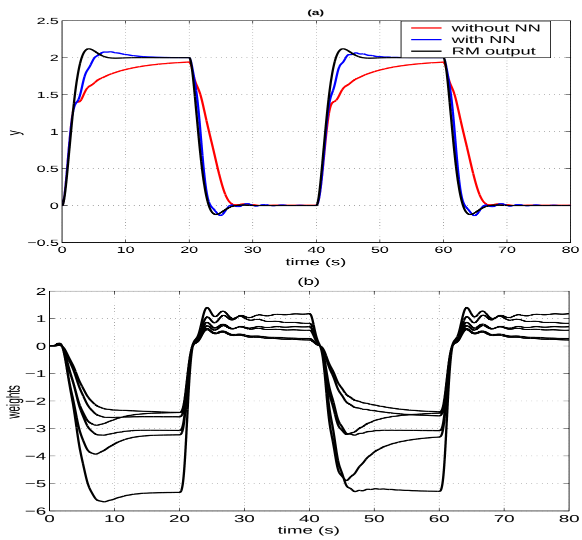

Figure 2: Simulation results: a) System response; b) NN weight evolution.

6 Conclusions

This paper has addressed the problem of contraction mapping assumption required in a recently developed approach for nonlinear adaptive output feedback control using neural networks. It was shown that the same update law that was obtained in the original paper can still be applied without the contraction mapping assumption provided that some additional knowledge about the control effectiveness is known by the designer. The result was illustrated on a simple example where the contraction mapping assumption can be violated if most of the current approaches were used to approximate the control effectiveness term.

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